

# Supervisory Control Theory;

A classical AI approach to build reliable systems

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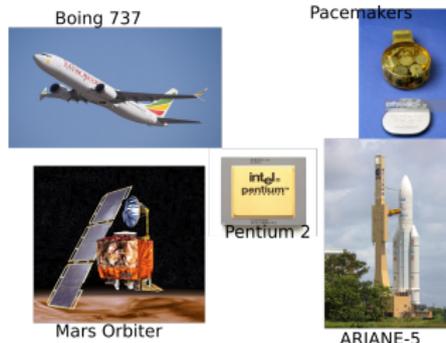
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# Outline

- 1 Introduction
- 2 Proofs, Verification and Synthesis
- 3 Supervisory Control
- 4 Building Reliable Systems
- 5 Applications in the real world
- 6 Problems
- 7 Conclusion

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# Past mistakes



- Boeing 747 software bugs<sup>1</sup>
- ARAINE 5 guidance error<sup>2</sup>
- Mars orbiter metric system error<sup>3</sup>
- Intel pentium fdiv bug<sup>4</sup>
- Lack of specifications for pacemakers

<sup>1</sup><https://www.nytimes.com/interactive/2019/business/boeing-737-crashes.html>

<sup>2</sup><https://www.bugsnap.com/blog/bug-day-ariane-5-disaster>

<sup>3</sup><https://www.bugsnap.com/blog/bug-day-mars-climate-orbiter>

<sup>4</sup>[https://en.wikipedia.org/wiki/Pentium\\_FDIV\\_bug](https://en.wikipedia.org/wiki/Pentium_FDIV_bug)

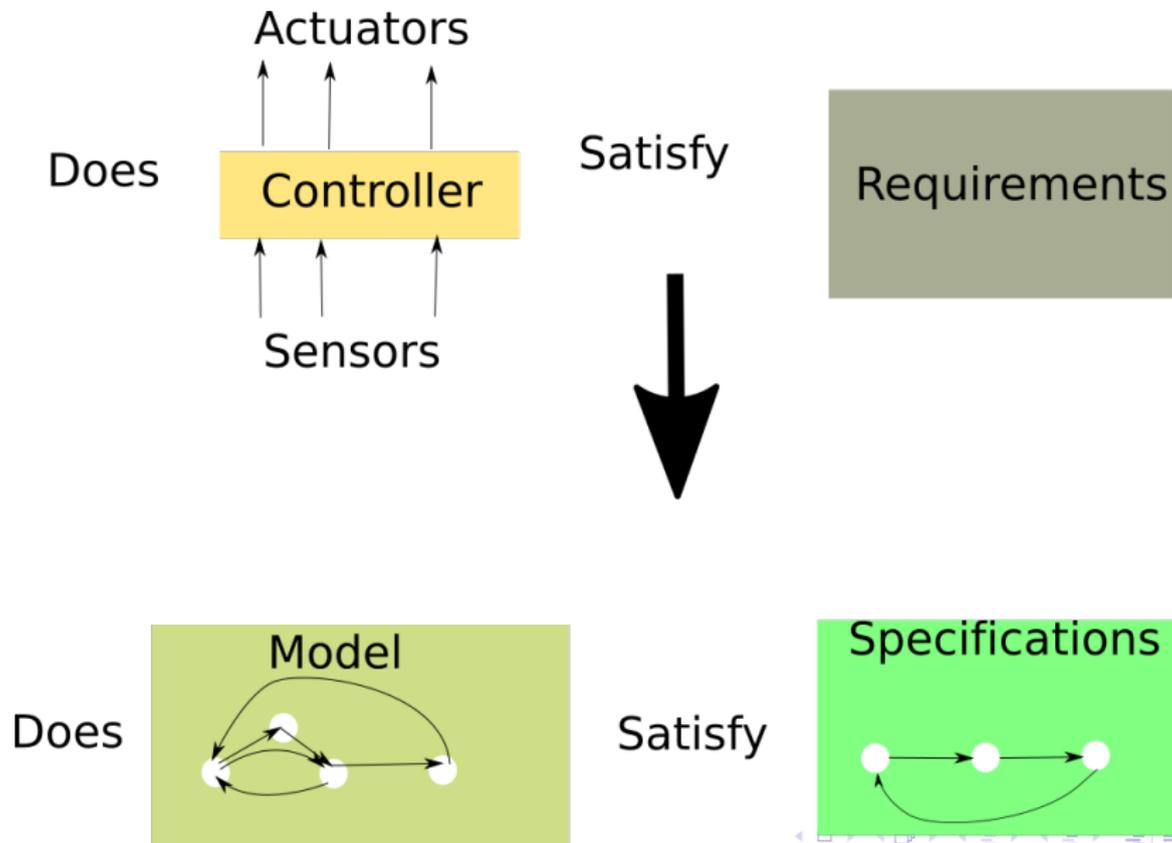
## Model

Description of the behavior of the given system defined mathematically.

## Specification

Mathematical description of the requirements pertaining to the model of the system.

# Model-based design



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Given a program and a contract.

**Goal:** Does the program fulfill the contract?

```
\programVariables {  
  int x;  
  int y;  
  boolean b;  
}
```

```
\problem {  
  \<{ if (b) { x = 1; } else { x = 2; } y = 3; }\> y > x  
}
```

---

<sup>5</sup><https://www.key-project.org/>

# Deductive Proofs

The screenshot displays a proof assistant interface with a toolbar at the top containing icons for play, Run Z3, a gear, a refresh, a scissors, a folder, a circular arrow, a pencil, a magnifying glass, and a document. Below the toolbar is a tabbed interface with four tabs: **Proof**, **Goals**, **Proof Search Strategy**, and **Info**. The **Proof** tab is active, showing a **Proof Tree** with the following nodes:

- 0:inEqSimp\_gtToGeq
- 1:polySimp\_mulComm0
- 2:inEqSimp\_sepPosMonomial1
- 3:polySimp\_mulComm0
- 4:polySimp\_rightDist
- 5:mul\_literals
- 6:polySimp\_mulLiterals
- 7:polySimp\_elimOne
- 8:if (b) { x=1; } else { x=2; }
- if b true
  - 9:x=1;
  - 22:{ }
  - 23:y=3;
  - 24:One Step Simplification: 2 rules
    - 25:{ }
  - 26:One Step Simplification: 7 rules
    - 27:add\_literals
    - 28:qeq\_literals
    - 29:closeTrue
    - 30:Closed goal
- if b false
  - 10:One Step Simplification: 1 rule
    - 11:notLeft
    - 12:x=2;
    - 13:{ }
    - 14:y=3;
  - 15:One Step Simplification: 2 rules

# Deductive Proofs

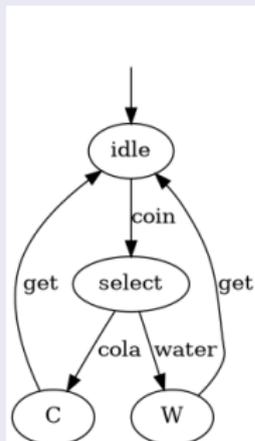
- Contract based systems
- Example Ethereum
- Develop and enforce legal contracts based on blockchain.

# Verification

Given a model of a system and specifications of the system.

**Goal** check if all specifications are satisfied by the model.

## Model



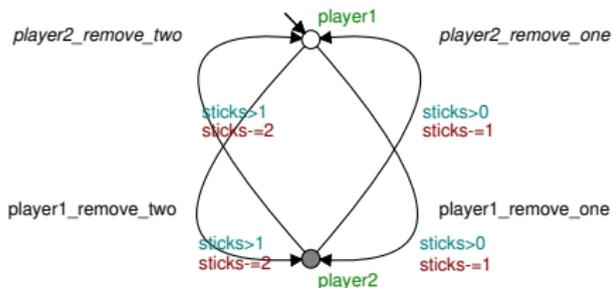
## Specification

"The vending machine only delivers a drink after providing a coin"

# Synthesis

Given a model of a system and specifications of the system.

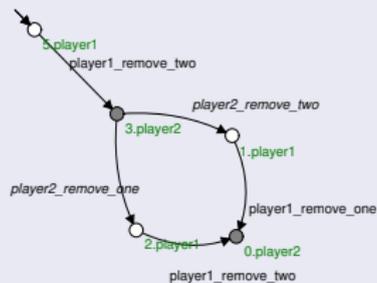
**Goal:** Generate a supervisor that restricts the behavior of the system to satisfy all the specifications.



## Specification

"Player one wins"

## Supervisor



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# Introduction

- Developed by Peter Ramadge and Walter Wonham in the late 80's <sup>6</sup>
- Control of DES
- Has been confined within academia for 30 years now.

## Automaton

$$G = \langle Q, \Sigma, \delta, q_0 \rangle$$

- $Q$  is the set of *states*
- $\Sigma$  is the *alphabet* containing the events,  $\Sigma = \Sigma_c \cup \Sigma_u$
- $\delta: Q \times \Sigma \rightarrow Q$  is the partial *transition function*
- $q_0 \in Q$  is the *initial state* of the system

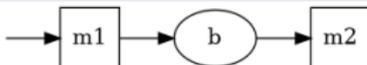
## Language

The language  $\mathcal{L}(G)$  is the set of all valid combinations of the alphabet in the automaton

<sup>6</sup>Ramadge, Peter J.; Wonham, Walter M. (January 1987). "Supervisory Control of a Class of Discrete Event Processes". *SIAM Journal on Control and Optimization*. 

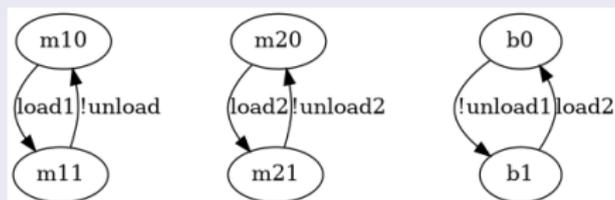
# Machine Buffer Machine

## Product flow



## System Behavior

$$\Sigma = \{load1, load2, !unload1, !unload2\}$$



$$\mathcal{L}(m1) = \{\epsilon, load1, load1.!unload1, load1.!unload1.load2, \dots\}$$

## Controllability

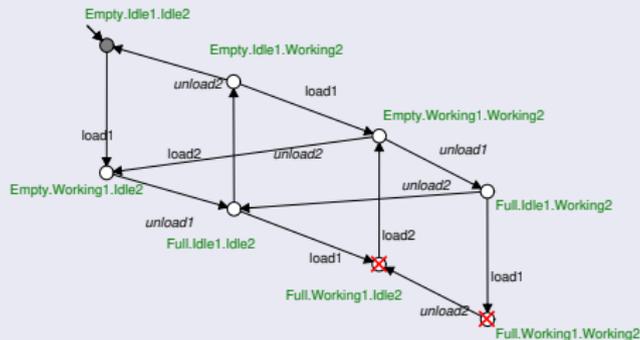
- Only some parts of the system can be controlled.
- Given a system, its specification, and the supervisor: the supervisor should always be able to control the desired outcome of the system to ensure specifications are met.

## Non-blocking

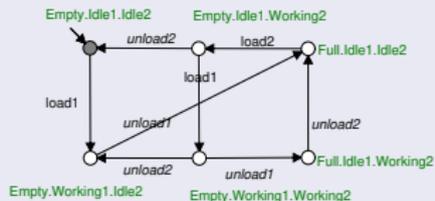
- At no point is the system stuck in one place with no possibility to reach its desired state.

# Machine Buffer Machine

## Synchronization



## Supervisor



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## A general approach towards developing reliable systems:

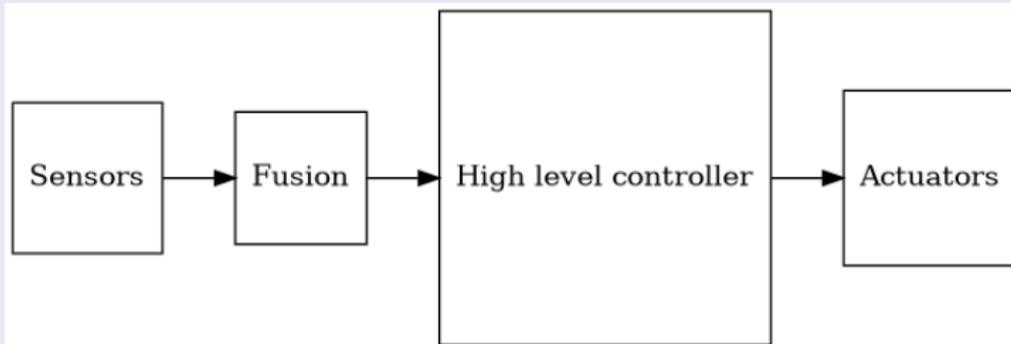
- 1 Pick one or more techniques that are relevant for your application.
- 2 Develop the model of the intended behavior of the system.
- 3 Translate the requirements of the project into specifications.
- 4
  - Verify and update the model till all specifications are satisfied.
  - Generate a supervisor for your system.
- 5 Tune and refine points 2,3 till the results are satisfactory.
- 6 Either implement the controller according the model or use automatic code generation if available.

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# Verification of lane change algorithm<sup>7</sup>

- Keeps track of state of execution
- Performs indication and lane change
- Cyclic execution

## System overview



<sup>7</sup>A. Zita, S. Mohajerani and M. Fabian, "Application of formal verification to the lane change module of an autonomous vehicle," 2017 13th IEEE Conference on Automation Science and Engineering (CASE), Xi'an, 2017

# Verification of lane change algorithm

## Modeling

- Modeled the system according to the code
- $1.4 \times 10^5$  states

## Specification



## Verification

- ~3Million states
- ~1Million were unreachable states
- An issue was found where the system does not behave as specified.
- The specification defines that the requested direction to turn, must be the direction the vehicle turns to.

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## State space explosion

- Number of states grows exponentially.
- The problem is known to be NP-Hard

## How to (and who) create(s) models?

- Defining models is an arduous task.
- Models are abstractions of reality.

## Where do specifications come from?

- Unclear on how to create all specifications
- Possibility of human errors in translating Natural language to formal spec

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- Automata are one way of modeling DES.
- They allow us to leverage on mathematical techniques to guarantee reliability of systems.
- Can be used to verify systems or generate a supervisor/controllers that satisfy given requirements.
- Suffer from state-space explosion problems.
- Lack of models and obtaining accurate models is a challenge.

# Agv system

- 4 AGV's that transport material
- 4 interacting zones

